

## GEOMETRIC EFFICIENCY OF EXPERIMENTAL SET-UPS IN THE REGION OF VERY HIGH VALUES OF MULTIPLICITY

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The processes of multiple production at multiplicity much larger than the average one are investigated. The description of such processes in terms of averaged with some weight function "local" characteristics is proposed. The problem of the possibility for the weight function to be determined in real experiments is investigated. Predictions of the geometric model and the model using a quantum-optical analogy are considered.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Геометрическая эффективность экспериментальных  
установок в области очень больших значений  
множественности

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Исследуются процессы множественного рождения при значениях множественности, много больших среднего значения. Предложено описание процессов при помощи усредненных с некоторым весом "локальных" характеристик. Исследуется проблема возможности определения весовой функции в реальных экспериментах. Рассмотрены предсказания геометрической модели, а также модели, использующей квантово-оптическую аналогию.

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1. It is natural to describe processes of multiple production at multiplicity  $n$  much larger than the average one  $\bar{n}(s)$  in terms of averaged characteristics<sup>1/</sup>. (Like in thermodynamics where for the description of a multi-particle system one should know only the average kinetic energy of particles — the temperature that can be measured just by a simple thermometer). Therefore, it is sufficient for experimental set-ups to measure only necessary averages. An approach like that advances experimental studies of inelastic processes towards the region of large  $n^*$  ( $n/\bar{n}(s) \approx 5 \div 10$ , i.e.  $n \approx 100 \div 1000$ ).

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\*Some theoretical aspects of production processes of a very large number of hadrons at high energies are discussed in Appendix.

We propose to pass from the study of such "local" characteristics as cross sections  $\sigma_n(s)$ ,  $\partial^3 \sigma_n / \partial p_1^3$ ,  $\partial^6 \sigma_n / \partial p_1^3 \partial p_2^3$  and others to the corresponding quantities averaged with some weight in the vicinity of values of multiplicity  $n$ , momenta  $p_1, p_2, \dots$ , angles, etc. For instance, instead of  $\sigma_n$  it is reasonable at  $n \gg \bar{n}(s)$  to study

$$T(n) = \sum_k \phi_k(n) \sigma_k(s), \quad (1)$$

where the weight function  $\phi_k(n)$  has maximum at  $k = n$  and is determined from a given experiment.

Under this statement of the problem, the efficiency of an experimental set-up obviously becomes very important. We shall discuss below a concrete problem: To what extent the fact that experimental set-ups cannot have  $4\pi$ -geometry is essential for the proposed statement of the problem. In other words, we aim at studying the problem of how well would the weight function  $\phi_k(n)$  be determined in a real experiment. For this purpose, in sections 2 and 3 we shall consider predictions of the geometric model <sup>/2/</sup> and of the model using a quantum-optic analogy <sup>/3/</sup> to determine losses of particles not detected experimentally.

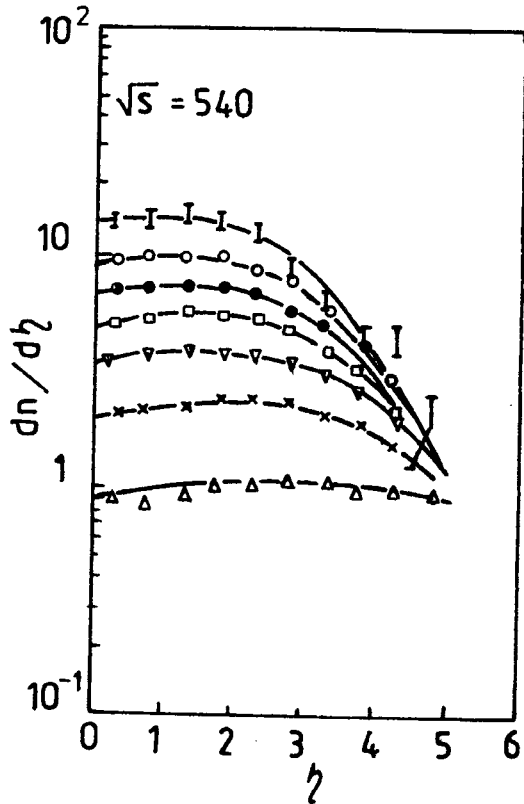


Fig.1.

- I —  $n_s > 71$
- o —  $n_s = 51-71$
- —  $n_s = 45-50$
- —  $n_s = 31-44$
- ▽ —  $n_s = 21-30$
- x —  $n_s = 11-20$
- Δ —  $n_s \leq 10$

2. To describe the angular distribution of produced particles we shall use the geometric model<sup>/2/</sup> well describing single-particle inclusive distributions in a wide region of energies (up to  $\sqrt{s} = 540$  GeV) and multiplicities (up to  $z \equiv n/\bar{n} = 3.5$ ), see fig.1. In this model

$$\frac{1}{\sigma_n} \frac{\partial^3 \sigma_n}{\partial p^3} = \frac{K}{\epsilon} e^{-\alpha P_{\perp}} e^{-\epsilon/T}, \quad (2)$$

where  $\epsilon = \sqrt{p^2 + m^2}$  is the energy of a produced particle and  $P_{\perp}$  is its transverse momentum. Dependence on the primary center of mass energy  $\sqrt{s}$  and multiplicity  $n$  is determined by the "temperature"  $T = T(s, n)$  of the gas of produced particles, the cutoff with respect to the transverse momentum  $\alpha = \alpha(s, n)$  and the normalization coefficients  $K = K(s, n)$ . By definition:

$$\int d^3 p \frac{1}{\sigma_n} \frac{\partial^3 \sigma_n}{\partial p^3} = n, \quad (3)$$

where  $n$  is the total number of produced particles if the integral in (3) is taken over the whole volume. Figure 2 shows the distribution of particles over pseudorapidity  $\eta = -\ln \tan \theta / z$ , following from (2), at  $\sqrt{s} = 6$  TeV,  $\sqrt{s} = 16$  TeV, and  $n = (1 \div 10)\bar{n}(s)$ .

Knowing  $T$ ,  $\alpha$ , and  $K$  we can, using the definition (3), find the number of particles that fell down into the cone with the aperture  $2\delta$  (all the calculations are made for the center of mass system, see fig.3) as a function of the total number of particles  $n$  and energy  $\sqrt{s}$ :

$$\begin{aligned} n_{\theta \leq \delta} &= \int_0^{\delta} \sin \theta d\theta \int_0^{2\pi} d\phi \int_0^{\infty} p^2 dp \frac{1}{\sigma_n} \frac{\partial^3 \sigma_n}{\partial p^3} = \\ &= \frac{2\pi K}{\alpha^2 - \beta^2} \left[ \frac{\alpha}{\sqrt{\alpha^2 - \beta^2}} \ln \frac{(\alpha + \beta + \sqrt{\alpha^2 - \beta^2})(\alpha + \beta - \sqrt{\alpha^2 - \beta^2} \tanh \eta_0 / 2)}{(\alpha + \beta - \sqrt{\alpha^2 - \beta^2})(\alpha + \beta + \sqrt{\alpha^2 - \beta^2} \tanh \eta_0 / 2)} \right. \\ &\quad \left. - \frac{\alpha + \beta e^{-\eta_0}}{\alpha + \beta \cosh \eta_0} \right], \end{aligned} \quad (4)$$

where  $\beta = 1/T$  is the average number of produced particles, and the maximally measurable pseudorapidity

$$\eta_0 = -\ln \tan \delta / 2. \quad (5)$$

Fig.2.

- 1 -  $n/\bar{n} = 10$
- 2 -  $n/\bar{n} = 7$
- 3 -  $n/\bar{n} = 5$
- 4 -  $n/\bar{n} = 3.4$
- 5 -  $n/\bar{n} = 2.5$
- 6 -  $n/\bar{n} = 1$

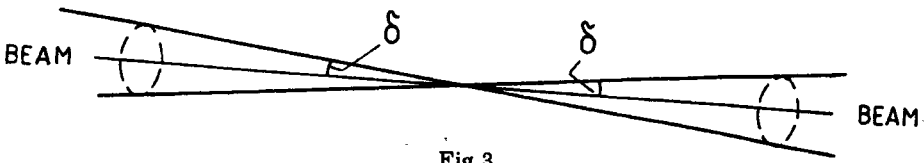
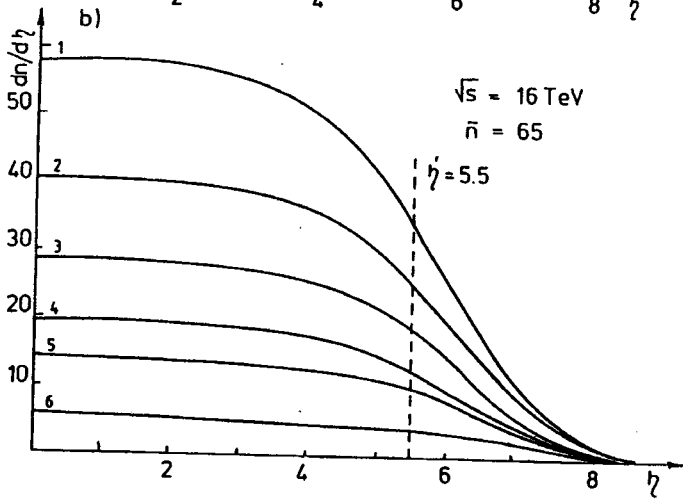
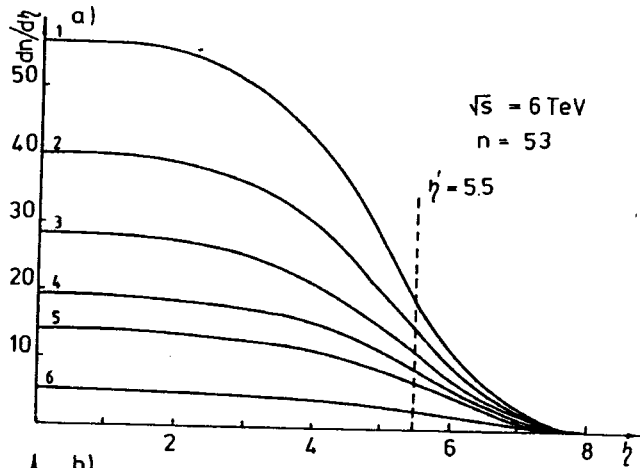


Fig.3

To determine  $T$ ,  $\alpha$ , and  $K$  we used extrapolation of experimental data at  $\sqrt{s} = 540 \text{ GeV}$  and  $z < 3.5$  into the region of large energies and multiplicities. The procedure of extrapolating requires introducing an additional parameter — the inelasticity coefficient at given multiplicity  $h$ . As a result, we have found the dependence

$$n_{\theta \leq \delta} = n_{\theta \leq \delta}(n, s, \delta, h).$$

see Figs. 4a,b.

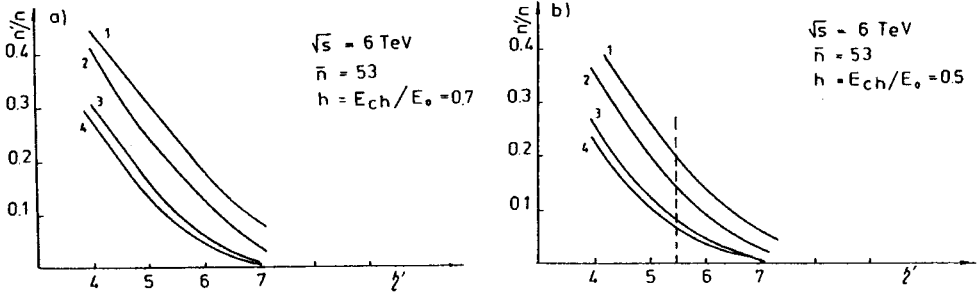


Fig. 4a, b

1	$n/\bar{n} = 2.5$	$n = 133$
2	$n/\bar{n} = 4$	$n = 212$
3	$n/\bar{n} = 8$	$n = 424$
4	$n/\bar{n} = 10$	$n = 530$

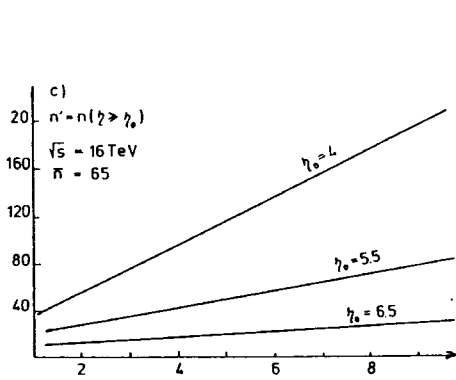


Fig. 4c

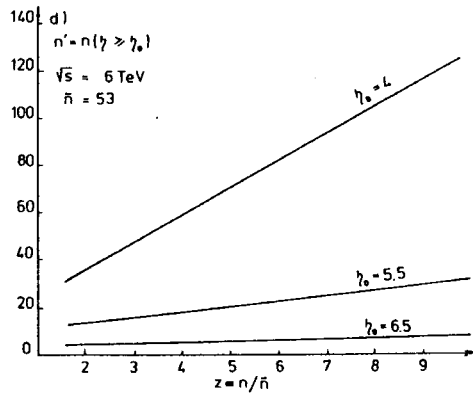


Fig. 4d

Figures 4c,d show the dependencies  $n_{\theta \leq \delta}$  on  $n$  for different values of  $\sqrt{s}$  and  $\delta$  under the assumption<sup>2/</sup> that the inelasticity coefficient  $h$  depends only on  $z = n/\bar{n}$  linearly.

Using (2) we can also find the portion of energy carried away by  $n_{\theta \leq \delta}$  particles:

$$\frac{E_{\theta \leq \delta}}{E_0} = \frac{z}{3} h \left[ 1 - \frac{\beta}{\alpha} \sinh \eta_0 \left( 1 + \frac{1}{\left( 1 + \frac{\beta}{\alpha} \cosh \eta_0 \right)^2} \right) \right]. \quad (6)$$

The numerical values of  $E_{\theta \leq \delta}/E_0$  are shown in Fig. 5.

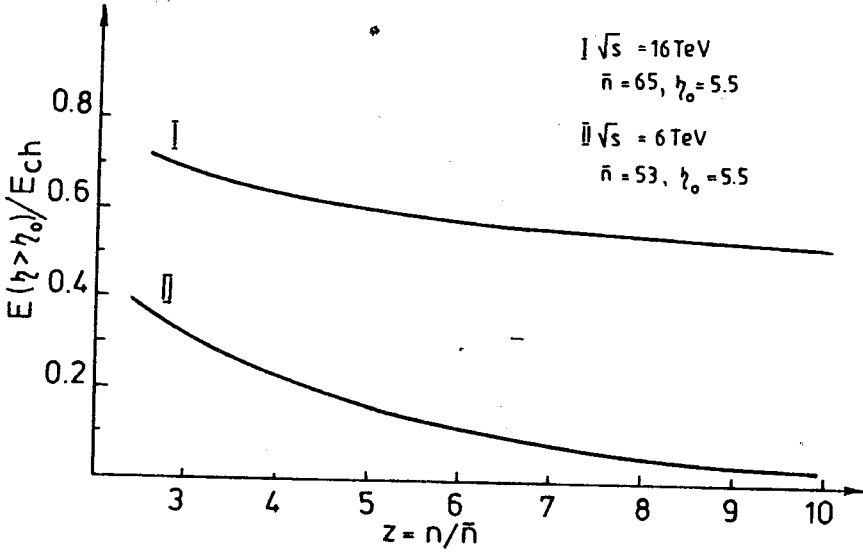


Fig.5

3. The above-made predictions concerning  $n_{\theta \leq \delta}$  are based on a certain procedure of extrapolating and, therefore, are model dependent. In what follows we shall give predictions based only on integral characteristics of inelastic interactions requiring no extrapolation. We shall use the fact that the negative binomial distribution for the probability of  $n$  particles to be produced

$$V(n) = \frac{(b+n-1)!}{(b-1)!n!} \left( \frac{b}{b+\bar{n}(s)} \right)^b \left( 1 + \frac{b}{\bar{n}(s)} \right)^{-n} \quad (7)$$

describes well experimental distributions over multiplicity of produced particles both in the full interval<sup>3/</sup> and limited intervals of rapidities<sup>4/</sup>.

Let us consider the known fact<sup>5,6/</sup> that the correlation length in the space of rapidities  $\Delta \eta \approx 2$ . Then, to estimate the number of particles  $n_{\theta < \delta}$  produced in the interval  $|\eta| \geq \eta_0$  we may assume that the probability for producing  $n_1$  particles in the interval  $|\eta| < \eta_0$  and  $n_{\theta \leq \delta} = n - n_1$  particles in the interval  $|\eta| \geq \eta_0$  is the product of the corresponding probabilities (7).

Following this natural assumption we find that

$$n_{\theta \leq \delta} = \bar{n}_{\theta \leq \delta} \left( 1 + \frac{b_1}{\bar{n}_1} \left( 1 + \frac{\bar{n}_{\theta \leq \delta}}{b_2} \right) \right), \quad (8)$$

where  $b_1$  and  $b_2$  are determined through dispersions of the relevant distributions. It is known<sup>/5/</sup> that  $b_1$  weakly depends on energy ( $b_1 \approx 3$ ). As  $\bar{n}_1 \gg b_1$  and, by definition  $\bar{n}_1 \gg \bar{n}_{\theta \leq \delta}$ , it follows from (8) that

$$n_{\theta \leq \delta} \approx \bar{n}_{\theta \leq \delta} \ll \bar{n}_1. \quad (9)$$

Thus, for  $\sqrt{s} = 6$  TeV and  $\delta = 0.5^\circ$  ( $\eta_0 = 5.4$ ) we have  $\bar{n}_{\theta \leq \delta} \leq 15$  ( $\bar{n}_{\theta \leq \delta} / \bar{n} \leq 0.3$ ), for  $\sqrt{s} = 16$  TeV and again for  $\delta = 0.5^\circ$  we have  $n_{\theta \leq \delta} = 22$  ( $\bar{n}_{\theta \leq \delta} / \bar{n} \leq 0.3$ ).

Thus, relative losses of particles  $n_{\theta \leq \delta} / n$  at  $n \gg \bar{n}$  are negligibly small and decrease with increasing  $n / \bar{n}$ . This conclusion agrees with the prediction obtained above.

4. We will not discuss possibilities of the given experiment to produce information about the above calculated losses of particles  $n_{\theta \leq \delta}$  as they essentially depend on the explicit form of the weight function  $\phi_k(n)$  completely determined by concrete realization of the experiment.

We should like to emphasize that the above-obtained estimates show the decrease of relative losses of particles (and energy), falling down into the fixed narrow cone along the beam of incident particles, in sampling events with large multiplicity in the central region of rapidities. Therefore, the condition for a large number of particles to get into the central region of rapidities really serves as a trigger for selecting events with a very large total multiplicity.

The importance of the latter is even greater as a trigger like that selects events with the formation of a small number of very massive jets produced as a result of the decay of superheavy partons (t- or perhaps other independent heavy quarks, massive intermediate bosons, Higgs's, supersymmetric partners of ordinary particles, etc.).

## Appendix

To complete the presentation we should like to cite the most important problems that can be investigated in the region of large values of  $n$ .

It is known<sup>/7/</sup> that the contribution to  $\sigma_n$  from soft processes (with small  $P_\perp$ )  $\sigma_n^s$  rapidly decreases with increasing  $n$ :

$$\sigma_n^s < O(e^{-n}), \quad (10)$$

whereas the contribution of the hard component (with large  $P_\perp$ ) decrea-

ses more slowly:

$$\sigma_n^h = O(e^{-n}). \quad (11)$$

Therefore, one can expect the dominance of hard processes in the region of large  $n^*$  when

$$n \geq n_0(s) \approx \bar{n}(s) e^{\alpha_0 \bar{n}(s) / \bar{n}_j(s)} \left(1 + O\left(\frac{\bar{n}}{n}\right)\right), \quad (12)$$

see fig.6. Here  $\alpha_0 = O(1)$ ,

$$\bar{n}(s) = 2.88 - 0.0867 \ln s + 0.171 \ln^2 s \quad (13)$$

is the average multiplicity in soft processes<sup>/5.6/</sup> and

$$\bar{n}_j(s) = 2.7 + 0.06 e^{1.4\sqrt{\ln \sqrt{s}}} \quad (14)$$

is the average multiplicity in the jet of the mass  $\sqrt{s}$ <sup>/5/</sup>.

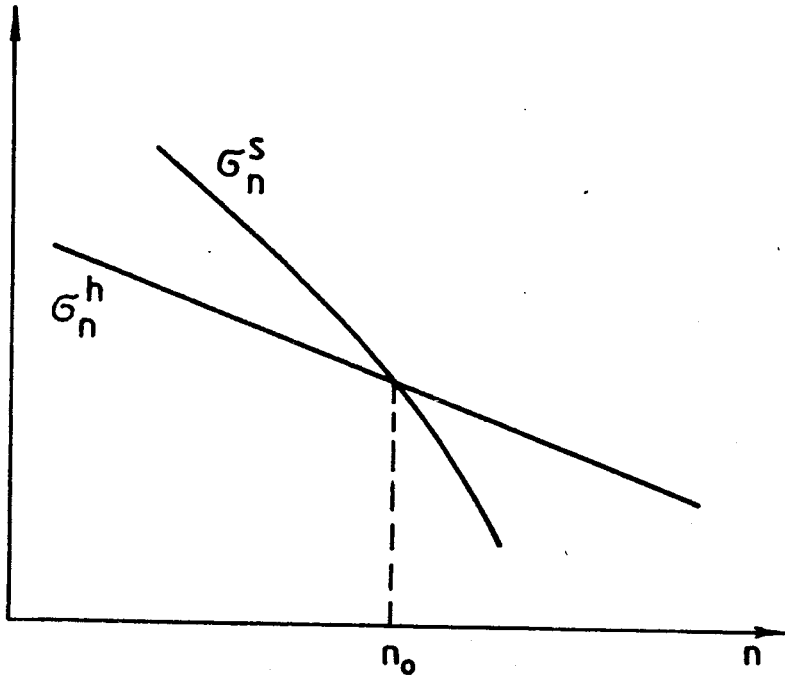


Fig.6

\*An indirect indication of this phenomenon is the violation the KNO scaling observed at the SSC-collider energies and the growth on the tail of distributions over multiplicity.



In order the region  $n > n_0$  would be experimentally accessible it is necessary, as it follows from (12), that

$$\bar{n}_j(s) > \bar{n}(s). \quad (15)$$

As is shown,  $(\bar{n}/\bar{n}_j) \sim 1$  in a wide region of energy values. Therefore, for attainable energies  $n_0 \approx (3 \div 5)\bar{n}$ .

As a result, in the region  $n > n_0$  one can expect restoration of the KNO scaling

$$\sigma_n(s) \approx \sigma_n^h(s) < e^{-\gamma n/\bar{n}_j(s)}, \quad (16)$$

where  $\gamma$  is determined only by the structure of the QCD Lagrangian.

In the transitional region  $n \sim n_0(s)$  the role of hard processes becomes more enhanced, which will be reflected in the appearance of "mini-jets" with small ( $\ll \sqrt{s}$ ) masses. In its turn, this should lead to the appearance of fluctuations in the density of produced particles in the limited intervals of rapidities. An indication of this phenomenon can be found in ref.<sup>/4/</sup>. Investigations in this region can shed light on collective phenomena in the nonequilibrium quark-gluon plasma<sup>/8/</sup>.

Since the distribution over multiplicity in jets is  $\sim e^{-n/\bar{n}_j(m)}$ , where  $m$  is the jet mass, at<sup>/9/</sup>

$$n > \frac{\bar{n}_j(s) \bar{n}_j(s/4)}{\bar{n}_j(s) - \bar{n}_j(s/4)} \equiv n_c(s) \quad (17)$$

there dominate processes with a small number of jets (and the distribution over multiplicity coincides with (16)). The numerical values of  $n_c(s)$  are given.

Thus,

- A.  $n < n_0(s)$  is the region of soft processes;
- B.  $n_0(s) < n < n_c(s)$  is the region of hard processes with the production of a large number of mini-jets (the transitional region — the region of the quark gluon plasma);
- C.  $n > n_c(s)$  is the region of hard processes with the production of a small number of massive jets.

The correlation analysis would show that  $n_c$  is the critical value of multiplicity at which the nature of correlations between partons becomes different. In other words, at  $n = n_c$  there occurs "structure phase transition" (transitions of that type can be observed only in non-equilibrium processes<sup>/9/</sup>).

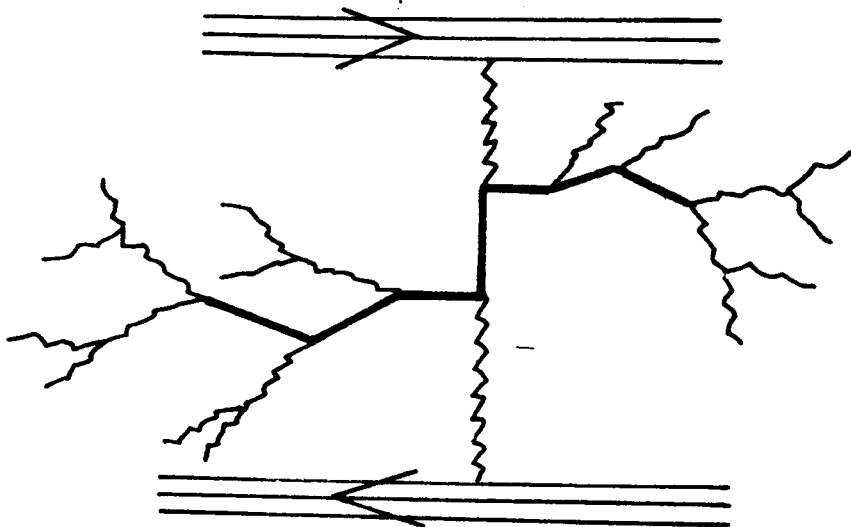


Fig.7

In the region  $n > n_c$  the number of jets should be minimal. This means that the mass of produced jets  $\simeq \sqrt{s}$  i.e. is very large. Hence, it follows that in the region  $n > n_c$ , at high energies jets should appear as a result of the decay of only heavy partons (see fig.7). Thus, in the region  $n > n_c$  one has a nice possibility of studying the production of heavy partons as the picture of their production at  $n > n_c$  cannot be masked by any background processes and almost the whole energy should be spent on their production.

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